



Effectiveness of problem-solving and student centred instructional strategies on students' achievement in algebra: A systematic review

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ABSTRACT

Algebra is central to advanced mathematical reasoning and provides a foundation for participation in science, technology, engineering and mathematics. Yet students at secondary and higher education levels continue to experience difficulty with symbolic representation, algebraic language, conceptual connections and the interpretation of unfamiliar problems. This systematic review examined the effectiveness of problem-solving and student-centred instructional strategies in improving students' achievement in algebra. The review was guided by the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA 2020) framework and drew on peer-reviewed empirical and review literature published between 2015 and 2025. Searches were conducted across Google Scholar, ERIC, ScienceDirect, SpringerLink, Taylor & Francis Online and relevant publisher databases using combinations of terms related to algebra achievement, problem-solving, inquiry, problem-based learning, flipped learning, blended learning, peer tutoring and active learning. Studies were retained when they addressed algebra or linear algebra instruction and reported outcomes related to achievement, conceptual understanding, engagement, mathematical thinking or problem-solving competence. A qualitative thematic synthesis identified five interconnected findings. First, structured problem-solving approaches strengthen strategic reasoning and support the translation of verbal information into algebraic representations. Second, student-centred strategies generally improve achievement and engagement when they are deliberately scaffolded. Third, many algebra difficulties are conceptual rather than merely procedural. Fourth, collaborative and active learning promote explanation, feedback and mathematical communication. Fifth, motivation, confidence, attitude and prior knowledge influence the effectiveness of instructional interventions. The review concludes that no single method is universally sufficient; effective algebra teaching requires a coherent combination of explicit guidance, purposeful exploration, collaborative activity, formative feedback and affective support. The article recommends sustained teacher professional development, curriculum alignment, improved access to learning resources and context-sensitive research on implementation quality and long-term learning outcomes.

Keywords: Algebra Achievement; Active Learning; Mathematics Education, Problem-Solving Strategies; Student-Centred Learning

I. INTRODUCTION

Algebra occupies a distinctive position in mathematics education because it enables learners to move from arithmetic calculation to generalised reasoning. Through algebra, students represent unknown quantities, identify patterns, formulate relationships, model change and communicate mathematical structures in concise symbolic form. These capabilities are not only required for success in advanced mathematics but are also important in science, technology, engineering, economics and many forms of everyday quantitative decision-making. For this reason, achievement in algebra is often treated as a gateway to later academic opportunity. Students who develop a strong conceptual foundation in algebra are better prepared to engage with functions, calculus, statistics, linear algebra and other mathematically demanding fields. Conversely, persistent difficulty in algebra may restrict progression across several areas of study and reduce learners' confidence in mathematics more broadly.

Despite its importance, algebra remains one of the areas in which many learners experience recurring and cumulative challenges. The difficulty is not explained simply by an inability to perform calculations. Students may carry out familiar procedures successfully while lacking a clear understanding of why those procedures work or when they should be applied. Chan et al. (2020) observed that learners commonly struggle with algebraic problem-solving because they must coordinate linguistic interpretation, symbolic representation, procedural knowledge and strategic decision-making. Giannakopoulos and Luneta (2023) similarly showed that success in linear algebra requires more than procedural fluency; learners need declarative, conceptual, procedural and strategic forms of knowledge that must work together. These findings suggest that low achievement may arise when instruction places excessive emphasis on memorising rules while offering insufficient support for meaning-making, reasoning and transfer.

The transition from arithmetic to algebra is especially demanding because familiar symbols and operations begin to serve more abstract purposes. A letter may represent an unknown value, a variable quantity, a generalised number or a parameter within a relationship. The equality sign must be understood relationally rather than merely as a signal to calculate an answer. Expressions may need to be interpreted structurally, and word problems require students to transform ordinary language into equations or inequalities. When these conceptual shifts are not explicitly addressed, students may develop fragile knowledge that works only in routine exercises. They may manipulate symbols without understanding them, select procedures based on superficial features or abandon unfamiliar tasks because they cannot identify a productive starting point.

Traditional teacher-centred instruction can be useful when a new procedure must be modelled clearly, but its limitations become visible when classroom activity is dominated by explanation, imitation and repetitive practice. In such settings, students may receive few opportunities to ask questions, compare strategies, test conjectures, justify solutions or learn from errors. They can become dependent on the teacher's demonstrated method and may be unable to adapt their knowledge when a problem is presented in a different form. This does not mean that direct teaching is inherently ineffective. Rather, it suggests that explanation and demonstration should be integrated with forms of activity that require students to think, communicate and apply ideas independently.

Problem-solving and student-centred instructional strategies respond to these concerns by positioning learners as active contributors to the learning process. Structured problem-solving approaches guide students to interpret the problem, identify relevant information, plan a solution, carry out the plan and evaluate the result. Problem-based learning begins with meaningful problems that motivate the acquisition and application of knowledge. Inquiry-based learning encourages students to investigate relationships, formulate conjectures and justify conclusions. Flipped and blended learning redistribute parts of instruction across individual preparation and classroom interaction, while peer tutoring and cooperative learning create opportunities for students to explain ideas and receive immediate feedback from classmates. Active learning more generally includes tasks such as group problem-solving, discussion, questioning, retrieval, representation and reflection.

A growing body of research indicates that these approaches can improve achievement, engagement and mathematical thinking. Ojaleye and Awofala (2018) found that blended learning and problem-based learning were associated with stronger achievement in senior secondary algebra than conventional instruction. Khasawneh et al. (2023) reported positive effects of inquiry-based learning in college algebra. Studies of flipped and collaborative instruction have also documented improvements in participation and learning in algebra and linear algebra contexts (Kirvan et al., 2015; Murphy et al., 2015). Peer tutoring has been shown to support mathematical thinking in linear algebra when learners are guided to discuss and explain their reasoning (Abdurrahman et al., 2020a, 2020b). At the university level, active learning strategies are associated with positive achievement outcomes across undergraduate mathematics courses (Lugosi & Uribe, 2022).

However, the evidence is distributed across different countries, educational levels, course structures and research designs. Some studies focus on secondary-school algebra, whereas others examine college algebra or linear algebra. Some interventions are technology-supported, while others depend mainly on classroom interaction. The outcomes measured also vary, including test performance, conceptual understanding, attitudes, engagement, persistence and mathematical thinking. These differences make it difficult for teachers and curriculum developers to determine what the evidence means for practice unless findings are brought together and interpreted systematically.

There is also a risk of treating student-centred instruction as a simple alternative to teacher-centred instruction. In reality, learner-centred strategies vary considerably in quality. An inquiry lesson without adequate guidance may leave students confused. A flipped classroom may reproduce passive learning if students watch lectures outside class and complete routine exercises inside class. Group work may be ineffective when roles are unclear or when participation is uneven. Peer tutoring can reinforce misconceptions if tutors are not prepared and monitored. The success of these approaches therefore depends on task design, teacher facilitation, sequencing, feedback, student readiness, access to resources and alignment with assessment. A systematic review must consequently examine not only whether a strategy appears effective, but also the conditions under which it is likely to support learning.

The present review addresses this need by synthesising research on problem-solving and student-centred instructional strategies in algebra education. It considers how these approaches influence achievement and engagement, how they respond to common conceptual difficulties and how cognitive and affective factors shape learning outcomes. The review also interprets the evidence through constructivist learning theory, which emphasises active knowledge construction, social interaction and guided support. By integrating findings across the literature, the article aims to provide a balanced account that is useful to teachers, teacher educators, curriculum developers, institutional leaders and researchers.

1.1 Statement of the Problem

Persistent low achievement in algebra remains a major concern in mathematics education. Students frequently experience difficulty in interpreting algebraic language, understanding symbolic notation, connecting multiple representations, selecting appropriate solution strategies and transferring knowledge to unfamiliar problems. These difficulties may continue even after students have practised standard procedures. As a result, successful completion of routine exercises does not always indicate deep understanding or readiness for advanced mathematical study. A central instructional problem is that algebra is often taught in ways that privilege speed and procedural accuracy over explanation, reasoning and conceptual connection. When lessons are dominated by worked examples followed by repetitive exercises, students may learn to imitate a method without understanding the underlying structure. Such knowledge is vulnerable because it can fail when the surface features of a problem change. Research has therefore called for instructional approaches that combine explicit teaching with opportunities for investigation, discussion, collaboration and reflection (Chan et al., 2020; Stewart et al., 2019).

Although many interventions have been proposed, including structured problem-solving models, problem-based learning, blended learning, flipped classrooms, inquiry-based learning, peer tutoring and active learning, their findings are often reported separately. The fragmented evidence base limits the ability of educators to compare strategies, identify shared mechanisms and recognise implementation conditions. In addition, some accounts celebrate learner-centred teaching without adequately considering the importance of scaffolding, teacher expertise, student readiness and contextual resources. A systematic synthesis is therefore needed to clarify the overall pattern of evidence and to identify implications for effective algebra instruction. This review responds to that problem by examining studies published between 2015 and 2025 that address problem-solving and student-centred strategies in algebra and linear algebra. It brings together evidence on academic achievement, conceptual understanding, mathematical thinking, engagement and affective outcomes. The intention is not to claim that one method is universally superior, but to determine how different approaches contribute to learning and what conditions make their use more educationally meaningful.

1.2 Research Objective

The objective of this study was to systematically review and synthesis empirical and review literature on the effectiveness of problem-solving and student-centred instructional strategies in improving students' academic achievement, conceptual understanding, engagement and mathematical thinking in algebra.

II. LITERATURE REVIEW

2.1 Theoretical Review

2.1.1 Constructivist Learning Theory

This review is anchored in constructivist learning theory. Constructivism views learning as an active process in which learners build and reorganise knowledge by relating new experiences to prior understanding. It rejects the assumption that knowledge can simply be transmitted intact from teacher to student. In algebra, this perspective is relevant because students must construct meaning for symbols, operations, equivalence, functions and abstract relationships.

The theoretical tradition draws on the work of Piaget, Vygotsky and Bruner. Piaget emphasised cognitive development through processes of assimilation and accommodation. Learners attempt to interpret new experiences through existing structures and modify those structures when they are inadequate. Vygotsky highlighted the social and cultural character of learning, especially the role of language, interaction and scaffolding within the zone of proximal development. Bruner emphasised discovery, representation and carefully structured support. Chand (2023) provides a contemporary discussion of these contributions and their continuing relevance to education. Applied to algebra, constructivism supports instructional practices in which students explore patterns, represent relationships, discuss strategies, justify claims and reflect on errors. It also clarifies the teacher's role. The teacher is not absent; rather, the teacher designs tasks, elicits prior knowledge, provides strategic guidance, asks questions, models expert reasoning when necessary and helps students connect informal ideas to formal mathematical concepts. This view avoids a false choice between direct instruction and discovery. Effective constructivist teaching often involves a deliberate sequence of exploration, guidance, explanation, practice and reflection.

The framework also explains why collaborative learning can be valuable. Through dialogue, students externalise their thinking and encounter alternative interpretations. Misconceptions become visible, and learners can appropriate more sophisticated ways of reasoning from peers and teachers. However, social interaction contributes to learning only when it remains mathematically focused. Constructivist theory therefore supports structured collaboration rather than unmonitored group activity. For this review, constructivism provides a lens for interpreting how problem-solving and student-centred strategies may influence achievement. The key mechanisms include activation of prior knowledge, cognitive conflict, meaningful representation, dialogue, scaffolding, reflection and gradual transfer of responsibility.

The framework also helps explain variation in outcomes: an intervention may underperform when tasks are poorly sequenced, guidance is insufficient, interaction lacks mathematical substance or students are not supported to consolidate their discoveries.

2.2 Empirical Review

2.2.1 Conceptualising Achievement in Algebra

Achievement in algebra is often measured through examination scores or researcher-developed tests, but a broader interpretation is necessary. Algebraic competence includes procedural fluency, conceptual understanding, strategic competence, adaptive reasoning and productive disposition. A student may manipulate an equation correctly yet remain unable to explain the meaning of the symbols, represent the same relationship graphically or recognise when the procedure is applicable. For this reason, achievement should be interpreted as a multidimensional outcome that includes accuracy, understanding, transfer, reasoning and confidence.

Linear algebra research reinforces this multidimensional view. Stewart et al. (2019) identified recurring themes in the teaching and learning of linear algebra, including the challenge of connecting abstract concepts, representations and formal definitions. Giannakopoulos and Luneta (2023) further emphasised the interaction of declarative, procedural, conceptual and strategic knowledge. These perspectives show why interventions that improve short-term test scores may still be insufficient if students remain unable to reason with concepts or apply them in new situations.

The distinction between procedural and conceptual knowledge is therefore central to the present review. Procedural knowledge includes rules, algorithms and sequences of actions, whereas conceptual knowledge concerns relationships, meanings and structures. The two forms of knowledge are mutually supportive rather than competing. Clear procedural instruction can reduce cognitive load and provide efficient tools, while conceptual understanding helps students choose, adapt and justify those tools. Effective algebra pedagogy should cultivate both, supported by opportunities for strategic problem-solving and reflection.

2.2.2 Problem-Solving Instruction in Algebra

Problem-solving instruction treats mathematical problems as vehicles for developing reasoning rather than as exercises completed only after teaching has occurred. A structured model, such as the stages commonly associated with Polya, encourages students to understand the problem, devise a plan, carry out the plan and look back at the result. George and Enefu (2019) reported that structured problem-solving models supported performance in simple linear equations, suggesting that explicit attention to the problem-solving process can reduce random trial-and-error and help learners organise their reasoning. The educational value of structured problem-solving lies in making normally hidden thinking processes visible. Students are prompted to identify what is known and unknown, translate words into mathematical relationships, select a representation, monitor progress and evaluate whether an answer is reasonable. These activities are especially important in algebraic word problems, where linguistic comprehension and symbolic representation must be coordinated. Chan et al. (2020) showed that students' difficulties arise at several points in this process, including understanding the problem, representing information and applying appropriate strategies.

Problem-solving before formal instruction can also be productive when it is carefully designed. Baumgartner et al. (2025) examined problem-solving before instruction in university linear algebra and demonstrated the potential of preparatory problem-solving to create awareness of conceptual gaps and make subsequent explanation more meaningful. This approach does not assume that students will independently discover complete formal knowledge. Instead, an initial problem can activate prior understanding, expose misconceptions and prepare learners to notice important features during later instruction. Nevertheless, problem-solving instruction is not automatically effective. Problems that are too difficult may overwhelm learners, while tasks that are too routine may fail to stimulate reasoning. Teachers must select tasks that are accessible yet challenging, sequence them carefully and provide prompts without taking over the thinking process. Feedback should address both the correctness of the answer and the quality of the reasoning. Students also need opportunities to compare methods and explain why a strategy is valid.

2.2.3 Student-Centred Instructional Strategies

Student-centred learning is an umbrella concept covering approaches in which learners actively engage with content, make decisions, communicate ideas and take increasing responsibility for learning. In algebra education, relevant strategies include problem-based learning, inquiry-based learning, blended learning, flipped instruction, peer tutoring, cooperative learning and other forms of active learning. These approaches differ in structure, but they share an intention to move students beyond passive reception.

Problem-based learning organises instruction around complex problems that require students to identify what they know, determine what they need to learn and apply new knowledge to propose a solution. In algebra, this may involve modelling real situations, analysing patterns or using equations to represent relationships. Ojaleye and Awofala (2018) found that problem-based and blended learning strategies improved achievement in senior secondary algebra.

The likely strength of problem-based learning is that it gives mathematical ideas a purpose and encourages integration of knowledge. Its main challenge is the need for careful facilitation so that the mathematical concepts remain explicit rather than becoming hidden behind the context.

Inquiry-based learning invites students to investigate mathematical relationships through questions, examples, conjectures and justification. Khasawneh et al. (2023) found that inquiry-based learning produced positive effects in college algebra compared with traditional lecture-based instruction. Inquiry can deepen understanding because students examine why patterns occur and how representations are connected. However, inquiry is most effective when teachers provide structured guidance, purposeful questions and opportunities to consolidate findings in formal mathematical language. Blended and flipped learning combine face-to-face interaction with digital or independent learning activities. In a flipped model, students encounter introductory material before class and use classroom time for application, discussion and problem-solving. Kirvan et al. (2015) demonstrated the value of flipping an algebra classroom focused on systems of linear equations, while Murphy et al. (2015) reported favourable performance and attitudes in a collaborative flipped linear algebra course. These approaches can increase time for active work, but they depend on students' preparation, access to materials and the quality of in-class tasks.

Peer tutoring and cooperative learning use social interaction as a learning resource. Students explain concepts, question one another, compare strategies and provide feedback. Abdurrahman et al. (2020a, 2020b) developed and examined peer tutoring strategies intended to improve mathematical thinking in linear algebra. The act of explaining can benefit both tutor and tutee because it requires ideas to be organised and expressed clearly. Yet peer learning requires defined roles, suitable group composition, teacher monitoring and a classroom culture in which errors are treated as opportunities for learning. Active learning is broader than any single named method. Lugosi and Uribe (2022) reviewed active learning strategies in undergraduate mathematics and identified positive effects on achievement. In algebra, active learning may include short problem-solving cycles, think-pair-share, multiple representations, retrieval questions, error analysis, student-generated examples, collaborative proofs and reflective writing. Its effectiveness depends on alignment: activities should be directly connected to the intended mathematical learning rather than used merely to create movement or participation.

2.2.4 Cognitive and Affective Influences on Algebra Learning

Instructional strategy alone does not determine achievement. Learners bring different levels of prior knowledge, language proficiency, working memory, motivation, confidence and attitudes to the classroom. These factors influence how they interpret tasks, respond to difficulty and persist when an initial strategy fails. Das and Ali (2023) found a relationship between attitudes towards algebraic problem-solving and achievement. Ganesen et al. (2020) likewise examined the relationship between learning styles and success in solving algebraic problems. Although learning-style claims should be interpreted cautiously, the study underscores the broader point that students engage with algebra in different ways and may require varied representations and supports.

Mathematics anxiety and low self-confidence can reduce participation and encourage avoidance. A student who expects to fail may abandon a problem quickly, hesitate to contribute during discussion or depend excessively on memorised rules. Student-centred learning can improve confidence by giving learners opportunities to succeed through collaboration and explanation, but it can also increase anxiety if tasks are insufficiently scaffolded or if classroom interaction exposes students to embarrassment. The emotional climate of the classroom is therefore part of instructional quality. Prior knowledge is equally important. New algebraic ideas are interpreted through existing mental structures. Misconceptions about equality, variables, negative numbers or operations can interfere with later learning. Effective instruction should therefore diagnose prior understanding and create opportunities for students to confront and revise misconceptions. Formative assessment, error analysis and targeted questioning are especially useful because they allow teachers to adjust support before difficulties become entrenched.

III. METHODOLOGY

3.1 Research Design

The study employed a systematic review design guided by the Preferred Reporting Items for Systematic Reviews and Meta-Analyses framework (PRISMA 2020) (Page et al., 2021). This design was appropriate because it enabled the researcher to identify, screen, assess and synthesize evidence from studies conducted across different educational contexts using a transparent and organized process. The review applied clearly defined eligibility criteria and examined findings in relation to recurring themes concerning problem-solving and student-centred instructional strategies in algebra.

A qualitative thematic synthesis was used instead of a statistical meta-analysis because the reviewed studies differed considerably in instructional approaches, educational levels, research designs, participant characteristics and outcome measures. While some studies reported achievement scores, others examined engagement, attitudes, conceptual

understanding and mathematical thinking. The thematic approach therefore allowed the researcher to identify common patterns while preserving contextual differences. The review was both descriptive and interpretive, as it examined not only reported outcomes but also the mechanisms and implementation conditions through which the instructional strategies influenced algebra learning.

3.2 Search Strategy

A structured literature search was conducted across Google Scholar, ERIC, ScienceDirect, SpringerLink, Taylor & Francis Online and relevant publisher websites. These sources were selected because they provide broad coverage of education, mathematics education and higher education research. Reference lists of relevant articles were also examined to identify additional studies. The search focused mainly on publications from 2015 to 2025 in order to capture recent developments while retaining influential studies relevant to contemporary algebra instruction.

Search terms were organised around three main areas: the mathematical domain, instructional approach and learning outcomes. Key terms included algebra, college algebra, linear algebra, problem-solving, problem-based learning, inquiry-based learning, student-centred learning, flipped classrooms, peer tutoring, active learning, achievement, conceptual understanding, engagement and mathematical thinking. Boolean operators were used to combine these terms through search strings such as “algebra AND problem-solving AND achievement” and “linear algebra AND peer tutoring.” The search process was iterative, beginning with broad searches before narrowing to specific strategies, educational levels, algebra difficulties and affective factors. Titles and abstracts were screened for relevance, after which potentially eligible studies were reviewed in full.

3.3 Inclusion and Exclusion Criteria

Studies were included if they focused on algebra, college algebra or linear algebra and examined problem-solving or student-centred instructional strategies. Studies addressing algebra learning difficulties and related learning processes were also considered when they contributed directly to understanding instructional effectiveness. Eligible publications reported outcomes such as academic achievement, conceptual understanding, mathematical thinking, engagement, attitude or problem-solving competence. Experimental, quasi-experimental, descriptive, mixed-methods, design-based and relevant review studies published in English were included, provided they contained sufficient methodological and bibliographic information for critical interpretation.

Studies were excluded if they were unrelated to mathematics education, focused only on mathematical content without educational component, lacked adequate methodological or findings information, or were based solely on unsupported opinion. Duplicate publications were also removed when a more complete version of the same study was available. Peer-reviewed and verifiable sources were prioritised, although the review acknowledges that database coverage and publication availability may have influenced study selection. Since the original manuscript did not provide numerical records for each PRISMA stage, the revised article reports the selection process transparently without inventing figures and recommends adding exact counts if the original search log is available.

3.4 Study Selection Process

Study selection followed the PRISMA stages of identification, screening, eligibility assessment and inclusion. Records were identified through database searches and reference-list checks, after which duplicate publications were removed. Titles and abstracts were then screened to determine whether the studies addressed algebra instruction, algebra learning difficulties or relevant problem-solving and student-centred strategies. Potentially relevant publications were subsequently reviewed in full text.

During full-text assessment, attention was given to the educational level, instructional focus, research design, reported outcomes and adequacy of methodological reporting. Studies that satisfied the inclusion criteria were retained for data extraction and synthesis, while borderline cases were judged according to their direct relevance to the review question. The screening process therefore emphasised conceptual relevance rather than keyword matching alone, ensuring that the final evidence base captured studies that meaningfully contributed to understanding teaching, learning and mathematical thinking in algebra.

3.5 Data Extraction

Data were extracted into a structured review matrix. The matrix recorded author and year, country or educational context where available, level of study, instructional strategy, research design, key outcome variables and principal findings. Additional notes captured implementation features such as scaffolding, collaboration, technology use, and feedback and teacher facilitation. The extraction process also distinguished between evidence of effectiveness and evidence about learning mechanisms. For example, an achievement gain was treated as an outcome, while increased opportunity for explanation or improved access to feedback was treated as a possible mechanism. This distinction supported a richer synthesis because similar outcomes can be produced through different instructional processes.

A further category captured limitations and contextual conditions. These included student access to digital resources, the need for pre-class preparation, variation in prior knowledge, teacher readiness and the risk of unequal participation in group work. Recording these factors helped prevent an overly general conclusion that any named student-centred strategy would be effective in all settings.

3.6 Data Analysis and Synthesis

The extracted evidence was analysed thematically by identifying recurring ideas across the reviewed studies. Initial codes focused on structured problem-solving, conceptual understanding, active participation, peer explanation, scaffolding, feedback, confidence and prior knowledge. These codes were subsequently organised into five broader themes: the contribution of structured problem-solving to algebraic reasoning; the influence of supported student-centred strategies on achievement and engagement; the conceptual and representational nature of algebra difficulties; the role of collaborative and active learning in strengthening explanation and feedback; and the effect of cognitive and affective factors on learning outcomes.

The themes were interpreted in relation to constructivist learning theory in order to explain how the identified instructional strategies supported learning. Because the included studies differed in design, context and quality, the synthesis adopted cautious language and avoided treating findings from a single study as universally applicable. Greater emphasis was placed on results supported across several studies or consistent with evidence on students' learning difficulties, while contradictory and conditional findings were retained and discussed to provide a balanced interpretation.

3.7 Quality Assurance and Ethical Considerations

Quality assurance was maintained through the use of multiple databases, explicit eligibility criteria, structured data extraction and systematic cross-checking of in-text citations against the reference list. Priority was given to peer-reviewed and verifiable sources with clear methodological information, while unsupported statistical generalizations and unverified PRISMA flow counts were avoided. Since the review relied entirely on published literature and did not involve human participants, formal ethical approval was not required. Nevertheless, scholarly integrity was upheld by accurately representing the included studies, clearly distinguishing evidence from interpretation, acknowledging the limitations of the review process and formatting all references in accordance with APA 7th edition guidelines.

IV. FINDINGS & DISCUSSION

4.1 Findings

The thematic synthesis produced five interconnected findings. The themes are presented together with their interpretation because the reviewer requested an integrated findings and discussion section. This structure allows the empirical pattern to be linked directly to theory, implementation and educational implications.

Table 1

Summary of selected evidence on instructional strategies in algebra

Study/source	Instructional focus	Context/outcome	Main implication
Ojaleye & Awofala (2018)	Blended learning and problem-based learning	Senior secondary algebra achievement	Learner-centred strategies improved achievement relative to lecture-dominated instruction.
Khasawneh et al. (2023)	Inquiry-based learning	College algebra achievement	Guided inquiry supported stronger achievement and conceptual engagement.
Kirvan et al. (2015); Murphy et al. (2015)	Flipped and collaborative instruction	Algebra and linear algebra learning	Pre-class preparation created more class time for application and discussion.
Abdurrahman et al. (2020a, 2020b)	Peer tutoring	Linear algebra mathematical thinking	Structured peer explanation supported reasoning and clarification of misconceptions.
Baumgartner et al. (2025)	Problem-solving before instruction	University linear algebra	Preparatory problem-solving made later instruction more meaningful.

Tashtoush et al. (2022)	Concept-rich instruction	Linear algebra achievement and attitudes	Attention to concepts and representations improved learning and attitudes.
Chan et al. (2020); Giannakopoulos & Luneta (2023)	Algebra learning difficulties	Problem interpretation and knowledge integration	Instruction must address conceptual, procedural and strategic knowledge.
Das & Ali (2023); Ganesen et al. (2020)	Affective and learner factors	Attitude, learning preferences and achievement	Motivation and confidence influence persistence and problem-solving performance.

4.1.1 Structured Problem-Solving Strengthens Algebraic Reasoning

The first theme shows that structured problem-solving strengthens students' ability to interpret, plan, execute and evaluate algebraic solutions. George and Enefu (2019) and Acquandoh et al. (2022) reported improved performance when learners were explicitly taught problem-solving procedures rather than expected to discover strategies through repeated examples alone. This is especially important in algebraic word problems, where students must identify relevant information, define unknown quantities and translate relationships into symbolic form. Chan et al. (2020) demonstrated that difficulties may occur during interpretation, representation or execution; therefore, structured routines such as restating the problem, identifying the unknown, selecting an appropriate equation and checking the reasonableness of the answer can expose misconceptions and prevent premature calculation.

Structured problem-solving also develops metacognitive and strategic competence by encouraging students to monitor progress, compare approaches and revise ineffective plans. Baumgartner et al. (2025) further showed that attempting a carefully designed problem before formal instruction can prepare learners to understand the value of subsequent explanations. However, productive struggle should be supported through prompts, worked examples, partial representations and peer discussion so that students remain responsible for reasoning without becoming overwhelmed. The findings also imply that assessment should reward the problem-solving process by allocating marks for interpretation, representation, justification, method selection and evaluation, rather than focusing only on the correctness of the final answer.

4.1.2 Student-Centred Strategies Improve Achievement and Engagement When Adequately Scaffolded

The second theme indicates that problem-based learning, inquiry-based learning, blended learning and flipped instruction can improve students' achievement and engagement when they are carefully designed and supported. Ojaleye and Awofala (2018) found that blended learning and problem-based learning enhanced achievement in senior secondary algebra, while Khasawneh et al. (2023) reported positive outcomes from inquiry-based learning in college algebra. These approaches strengthen learning by requiring students to explore patterns, apply concepts, discuss ideas and connect algebraic representations with meaningful situations. Problem-based learning develops conceptual understanding and strategic competence by engaging learners in identifying relevant information, constructing mathematical representations and evaluating solutions. Similarly, guided inquiry supports students in connecting tables, graphs, equations and verbal descriptions through purposeful questions, carefully selected examples and explicit consolidation of formal concepts.

Flipped and blended learning can create additional classroom time for problem-solving, interaction and immediate feedback, as demonstrated by Kirvan et al. (2015) and Murphy et al. (2015). However, their effectiveness depends on students' access to technology, preparation and the quality of learning materials. Pre-class resources should be concise, accessible and clearly connected to classroom activities, while digital tools should promote representation, feedback, reflection and collaboration rather than merely reproduce lectures and routine exercises. Overall, the reviewed evidence shows that student-centred teaching does not mean removing teacher guidance. The most effective approaches involve clear learning goals, carefully selected tasks, anticipation of misconceptions, structured prompts, active monitoring and consolidation of understanding. Thus, learner activity and teacher scaffolding should operate together to promote meaningful algebra learning.

4.1.3 Students' Difficulties in Algebra Are Predominantly Conceptual and Representational

The third theme indicates that students' difficulties in algebra are primarily conceptual and representational rather than purely computational. Chan et al. (2020) identified persistent challenges in algebraic problem-solving, while Giannakopoulos and Luneta (2023) and Stewart et al. (2019) showed that success in algebra and linear algebra requires learners to integrate procedural, conceptual and representational knowledge. Students may correctly manipulate equations or perform matrix operations while misunderstanding equality, variables, functions, vectors or transformations. These fragmented understandings explain why repeated procedural practice alone may not improve achievement. Concept-rich instruction is therefore essential because it connects procedures to underlying meanings,

makes relationships explicit and encourages learners to explain their reasoning. Tashtoush et al. (2022), for example, reported positive effects of concept-rich teaching on students' achievement and attitudes in linear algebra.

Effective algebra teaching should use multiple representations and provide learners with opportunities to move between verbal descriptions, numerical patterns, symbolic expressions, tables, graphs and contextual situations. Asking students to interpret equations, construct corresponding graphs and explain relationships in words can reveal misconceptions that symbolic manipulation may conceal. Error analysis can further deepen understanding by requiring learners to identify incorrect reasoning, explain the source of an error and propose a correction. These findings also have implications for curriculum pacing, as rapid content coverage may encourage superficial learning and prevent students from making meaningful connections. Teachers and curriculum developers should therefore prioritise depth of understanding and provide repeated attention to foundational concepts such as equality, variable, equivalence, function and representation.

4.1.4 Collaborative and Active Learning Strengthen Explanation, Feedback and Mathematical Communication

The fourth theme is that collaborative and active learning strengthen understanding by increasing opportunities for explanation, comparison and feedback. Peer tutoring and group problem-solving require students to express ideas that might otherwise remain implicit. Abdurrahman et al. (2020a, 2020b) showed how a peer tutoring strategy could be designed to support mathematical thinking in linear algebra. Lugosi and Uribe (2022) found positive effects of active learning strategies in undergraduate mathematics more broadly.

Explanation is a powerful learning process because it requires students to organise knowledge and make connections. A learner who can perform a procedure but cannot explain it may possess fragile understanding. During peer interaction, students must translate private reasoning into language that another person can follow. Questions from peers can expose gaps, while alternative methods can broaden strategic knowledge. The tutor also benefits because teaching a concept demands retrieval and clarification. Immediate feedback is another advantage. In a lecture-dominated environment, misconceptions may remain unnoticed until homework or examinations are marked. In collaborative problem-solving, students receive responses while they are still working. Peers can question an assumption, identify an error or suggest a representation. The teacher can listen to group discussions and intervene strategically. This creates a formative assessment environment in which instruction responds to learning as it occurs.

Collaboration also supports mathematical communication. Algebra is not only a set of procedures; it is a language for expressing relationships and arguments. Students need to use terms accurately, justify transformations and interpret representations. Group discussion provides practice in this disciplinary language. Over time, learners can move from informal explanations to more precise mathematical statements. Nevertheless, group work can fail when participation is unequal. Confident students may dominate, while others copy answers without engaging. Peer tutors may provide procedures without explanation or may reinforce misconceptions. Effective collaborative learning therefore requires structure. Teachers can assign roles, use individual accountability, rotate responsibilities and require each student to explain part of the solution. Group size should be manageable, and tasks should be designed so that interaction is necessary rather than optional. Active learning should likewise be judged by cognitive engagement, not visible activity alone. A classroom can appear lively while students remain focused on low-level tasks. The most valuable activities require learners to retrieve, represent, explain, justify, compare or evaluate. Teachers should ask what thinking an activity demands and how that thinking contributes to the intended algebraic concept.

4.1.5 Cognitive and Affective Factors Shape Achievement and Intervention Effectiveness

The fifth theme indicates that students' achievement in algebra is shaped by cognitive and affective factors, including prior knowledge, attitude, motivation, confidence and persistence. Das and Ali (2023) established an association between students' attitudes towards algebraic problem-solving and their achievement, while Ganesen et al. (2020) examined the relationship between learning styles and algebraic problem-solving performance. These findings suggest that the effectiveness of instructional strategies depends partly on what students already know and how they perceive their mathematical ability. Confident learners are more likely to attempt difficult problems, test alternative methods and respond positively to feedback, whereas students with repeated experiences of failure may avoid challenging tasks, guess answers or depend excessively on memorized procedures. Teachers should therefore use diagnostic assessment to identify differences in prior knowledge and misconceptions, then provide appropriate scaffolding and progressively challenging tasks that allow learners to experience meaningful success.

Effective algebra instruction should also create a supportive classroom environment that strengthens motivation, participation and confidence. Feedback should extend beyond indicating whether an answer is correct by recognizing effective strategies, explaining misconceptions and identifying specific areas for improvement. Although student-centred strategies can promote agency, interaction and positive attitudes, they may also cause anxiety when learners are required to participate publicly without adequate preparation. Teachers should therefore establish respectful classroom norms, provide sufficient thinking time and use varied participation structures that include both confident and less

confident learners. Overall, cognitive, social and emotional support should be integrated through conceptual scaffolding, strategic instruction, collaborative learning and confidence-building activities that activate prior knowledge, encourage peer discussion and support sustained engagement with algebraic problems.

4.1.6 Cross-Cutting Interpretation: No Single Strategy is Sufficient

Across the five themes, the evidence does not support the search for a single universally superior method. Each strategy offers particular strengths and faces particular risks. Structured problem-solving develops strategic control but can become formulaic if students merely memorise stages. Inquiry promotes conceptual exploration but may overwhelm learners when guidance is insufficient. Flipped learning can release class time for active work but depends on preparation and access. Peer tutoring strengthens explanation but requires monitoring and accountability. The most defensible conclusion is that effective algebra instruction combines complementary elements. Clear explanation is needed when students encounter unfamiliar notation or formal definitions. Purposeful problems are needed to apply and connect ideas. Collaborative activity is needed to support explanation and feedback. Individual practice is needed to develop fluency. Reflection is needed to consolidate learning and support transfer. Formative assessment is needed to identify misconceptions and adjust teaching.

This combination can be described as guided student-centred instruction. The term emphasises that students are active, but their activity is intentionally designed and supported. It also avoids an unproductive opposition between teacher-centred and student-centred pedagogy. Effective teachers move flexibly between modelling, questioning, facilitation, feedback and consolidation according to the learning goal and students' needs. The constructivist framework helps explain this synthesis. Learners construct meaning through active engagement with tasks and social interaction, but expert guidance shapes the quality of that construction. Scaffolding is gradually withdrawn as competence develops. The aim is not permanent dependence on the teacher, nor unsupported discovery, but increasing independence grounded in secure conceptual understanding. Context must also guide implementation. Schools and universities differ in class size, technology access, curriculum demands, and teacher preparation and student background. A strategy shown to be effective in a well-resourced college may require adaptation in a large secondary classroom. The core mechanisms active reasoning, representation, explanation, feedback and scaffolding may be more transferable than the outward form of a particular intervention.

4.1.7 Implications for Teaching Practice

For classroom practice, the review suggests that algebra lessons should begin with clear learning goals that identify both procedural and conceptual outcomes. A lesson on solving equations, for example, should specify not only that students will obtain correct solutions but also that they will understand equivalence, justify transformations and evaluate solutions. Activities and assessment can then be aligned with these outcomes.

Teachers should use carefully sequenced problems. Early tasks can activate prior knowledge and reveal misconceptions, while later tasks can increase complexity and require transfer. Students should be encouraged to represent problems in more than one way and to explain why a method works. Worked examples remain valuable, especially for novice learners, but they should be accompanied by questions that direct attention to underlying structure. Collaborative learning should be structured. Small groups can compare methods, analyse errors or solve different parts of a complex problem. Individual accountability can be maintained through brief written explanations, random reporting or follow-up questions. Teachers should circulate, listen for misconceptions and use selected student responses to organise whole-class discussion.

Technology should be used purposefully. Videos, simulations, graphing tools and online quizzes can support preparation, visualisation and feedback. Their use should be evaluated according to the mathematical thinking they enable. Access barriers should be anticipated, and equivalent alternatives should be available where necessary. Finally, classroom climate should support persistence. Students need time to think, opportunities to revise and feedback that identifies specific ways forward. Errors should be discussed respectfully. Such practices can reduce anxiety and help students see difficulty as part of learning rather than as evidence that they do not belong in mathematics.

4.1.8 Implications for Curriculum, Teacher Education and Policy

Curriculum documents should balance procedural fluency with conceptual understanding, reasoning and application. Overly dense syllabi can undermine student-centred teaching by forcing rapid coverage. Curriculum developers should identify essential algebraic ideas and provide guidance on representations, misconceptions and problem types. Assessment frameworks should reward explanation and strategic reasoning rather than memorisation alone.

Teacher professional development is essential because student-centred strategies demand sophisticated pedagogical judgement. Teachers need opportunities to design tasks, anticipate student thinking, facilitate discussion, use formative assessment and integrate technology. Professional learning should be practical and sustained rather than

limited to one-off workshops. Lesson study, peer observation and collaborative planning can help teachers test and refine approaches in their own contexts. Institutional leaders should support implementation through manageable class arrangements, access to learning resources and recognition of the time needed to prepare high-quality activities. In higher education, departments can develop shared problem banks, pre-class materials and assessment rubrics. In schools, mathematics teams can analyse student work and coordinate support for foundational concepts. Policy should avoid promoting named pedagogies as slogans without providing implementation support. A national or institutional directive to use inquiry, flipped learning or group work will not improve achievement unless teachers understand the approach and have suitable resources. Policy should therefore focus on the underlying principles of effective practice and provide mechanisms for professional learning, monitoring and adaptation.

V. CONCLUSION & RECOMMENDATIONS

5.1 Conclusion

This systematic review established that structured problem-solving and student-centred instructional strategies including problem-based learning, inquiry-based learning, blended learning, flipped instruction, peer tutoring, concept-rich teaching and active learning—can improve students' achievement, engagement, mathematical thinking and attitudes in algebra. These approaches are most effective when they are aligned with clear learning objectives and supported through purposeful scaffolding, appropriate resources and consistent teacher guidance. The findings further indicate that many difficulties in algebra are conceptual, representational and strategic rather than purely procedural. Students may apply rules correctly without understanding the relationships they represent. Effective instruction should therefore connect symbols, verbal explanations, graphs, tables and real-life contexts while providing opportunities for students to explain, justify and reflect on their reasoning. Procedural fluency should be developed together with conceptual understanding and strategic competence.

Collaborative and active learning can strengthen peer explanation, feedback and mathematical communication, but their success depends on careful lesson design, clear roles, teacher monitoring and equitable access to learning resources. The review therefore supports guided student-centred teaching, in which explicit instruction and active learning complement one another. It also emphasizes the importance of prior knowledge, confidence, motivation and attitudes, showing that improved algebra achievement requires coordinated attention to pedagogy, assessment, curriculum, teacher development and a supportive learning environment.

5.2 Recommendations

Teachers should integrate structured problem-solving into algebra lessons by guiding students to interpret problems, select appropriate strategies, justify their procedures and evaluate their answers. Algebra instruction should combine clear teacher explanation with purposeful exploration, discussion, application and reflection to strengthen conceptual understanding.

Schools and higher education institutions should support student-centred approaches such as problem-based learning, inquiry, peer tutoring, blended learning and flipped instruction with adequate scaffolding, resources and teacher preparation. Teachers should use multiple representations, collaborative tasks and assessment practices that reward conceptual understanding, mathematical reasoning, justification and the quality of the solution process. Teacher professional development and future research should focus on task design, formative assessment, misconception diagnosis, technology integration, implementation fidelity, long-term retention and adaptation across different educational contexts.

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